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Separation Prevention as an Indirect Problem Based on the Triple-Deck Theory

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The streamlining technique for preventing boundary-layer separation is modeled as an indirect problem based on the triple-deck theory. A numerical scheme that handles this problem with accuracy and speed is also formulated. As an application, the approach is used to design augmented compression corners that produce attached supersonic flows.

Introduction

BOUNDARY-LAYER separation is an undesirable phenomenon in both internal and external flow applications. In internal flows, separation reduces the efficiency of energy-exchange devices (turbines and compressors) because it is associated with loss of energy. The formation of separation bubbles in flow passages (pipes, diffusers, or nozzles) results in a smaller effective discharge area, thus altering the performance of the unit. In external flows, where minimum drag and maximum lift are required, separation with extended low-pressure wake results in large pressure drag. This extended separation takes place in flows past bluff bodies and in flows past stalled airfoils, in which case a disastrous drop in lift is also experienced. Local separation with a small separation bubble is no less dangerous because it may develop with unpleasant suddenness into extended separation.[1]

Boundary-layer control aimed at prevention of separation has been of primary concern to fluid mechanists. Several techniques have been employed; e.g., streamlining, suction, and acceleration.[2] Not only do these techniques prevent separation, but they also have a stabilizing effect tending to delay the transition of a laminar boundary layer to a turbulent one. As a consequence, additional drag reduction and energy savings are achieved.

The present paper is concerned with streamlining as a means of preventing small-bubble separation. The situation under consideration is that of a two-dimensional steady laminar supersonic flow past a compression corner (Fig. 1) of sharp angle that is large enough to produce a sizable separation bubble. By properly augmenting the corner, a fully attached flow can be obtained. More generally, we seek to find the corner shape that produces a prescribed minimum surface shear.

To model the flow, one has to realize that separation is a high-Reynolds-number effect. As the Reynolds number Re grows indefinitely, the flow past a compression corner (flat plate-ramp conjunction) develops, in the vicinity of the juncture point A, into a triple-deck structure,[3] provided that the corner angle is of order $Re^{-1/4}$. This structure facilitates a viscous/inviscid interaction that allows the oncoming flat-plate boundary layer upstream of A to adjust itself to the presence of the corner and to evolve into a ramp boundary layer downstream of A.

Based on the asymptotic triple-deck structure, two models have been applied to the direct (analysis) problem of finding the flow variables when the corner shape is given. The leading order model requires the numerical solution of the lower-deck equations only. Such solutions were obtained by Jenson et al.[4] They reveal that separation takes place when a normalized corner angle α exceeds a value of $\cong 1.65$, and a sizable separation bubble forms when α is greater than $\cong 3$. The composite model of the interacting boundary-layer equations allows some finite Reynolds-number effects to be retained. Comparisons with experimental measurements and Navier-Stokes calculations show that, whereas the former model is accurate at very high values of Re only, the latter is accurate at moderate to high values of Re. It is also noted that, for the same value of the normalized angle α, the extent of the separation bubble predicted by the lower-deck model is larger than that predicted by the interacting boundary-layer model whose prediction gets smaller as Re decreases.[5]

The model we choose for the present indirect (design) problem is that of the lower-deck equations rather than the interacting boundary-layer equations. The chosen model is simpler, being of incompressible nature. It requires a smaller computational domain because it excludes the main deck of the asymptotic structure. Experience with the analysis problem indicates that the numerical methods—that are apt to be iterative—converge faster when applied to the lower-deck equations. Furthermore, the fact that the lower-deck problem can be formulated in terms of universal variables that are rid of the basic flow parameters allows a solution obtained for a given normalized angle α to correspond to different combinations of flow parameters and physical angle α^*. [See Eq. (6c) for the relation between α and α^*.] On the other hand, a design for the value of α that corresponds to a given set of flow parameters and α^* would be more conservative when the lower-deck model is adopted because this model predicts a larger separation bubble. For these reasons, the lower-deck model is expected to be more computationally efficient and more suitable for design purposes.

The numerical approach we use to deal with the design problem is based on a two-sweep iterative scheme formulated by Napolitano et al.[6] to handle the corresponding analysis problem. By imposing the additional condition of prescribed minimum surface shear, the corner shape is determined nonit-

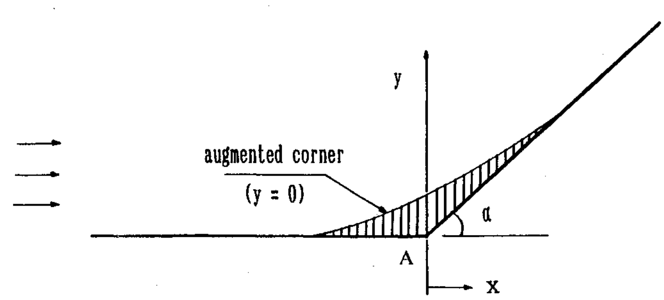

Fig. 1 Flow configuration.

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eratively; i.e., it does not require a new iteration loop. In fact, because the flow is now nonseparating, the design problem converges faster than the analysis problem. Smoothness of the augmented corner is guaranteed through the use of a finite difference representation that avoids averaging in the streamwise direction. However, some adjustments are needed so that an irregularity in the form of a bumpy augmentation front does not appear.

Statement of the Problem

The problem of steady laminar supersonic flow past the augmented compression corner shown in Fig. 1 can be reduced,[3] through asymptotic analysis in the limit of infinite Reynolds number, to the following lower-deck problem.

Let x measure the downstream distance from A along the flat plate and y measure the distance normal to the flat plate with $y = 0$ along the surface. The corresponding velocity components $u(x, y)$ and $v(x, y)$, and the shear function $w(x, y)$ are governed by the flow equations (differentiation is denoted by a subscript)

$$u_x + v_y = 0, \qquad u_y = w \qquad \text{(1a, 1b)}$$

$$w_y + uv_y - u_y v = Q \qquad \text{(1c)}$$

and satisfy the conditions

$$u = 0, \qquad v = 0 \qquad \text{at } y = 0 \qquad \text{(1d, 1e)}$$

$$w \to 1, \qquad u \to y - D + B \qquad \text{as } y \to \infty \qquad \text{(1f, 1g)}$$

$$u \to y \qquad \text{as } x \to -\infty \qquad \text{(1h)}$$

For a sharp corner, the surface function $B(x)$ takes the form

$$B = \alpha x H(x) \qquad \text{(2a)}$$

where α is the ramp angle and H is the Heaviside function [$H(x) = 0$ when $x < 0$ and $H(x) = 1$ when $x > 0$].

For an augmented corner, B can be written as

$$B = \alpha x H(x) + S \qquad \text{(2b)}$$

where $S(x)$ is the augmentation function that is expected to be piecewise continuous having nonzero values over a short distance around the corner A and zero value elsewhere.

The displacement function $D(x)$ is related to the pressure $P(x)$ through the supersonic interaction law

$$P = D_x \qquad \text{(3a)}$$

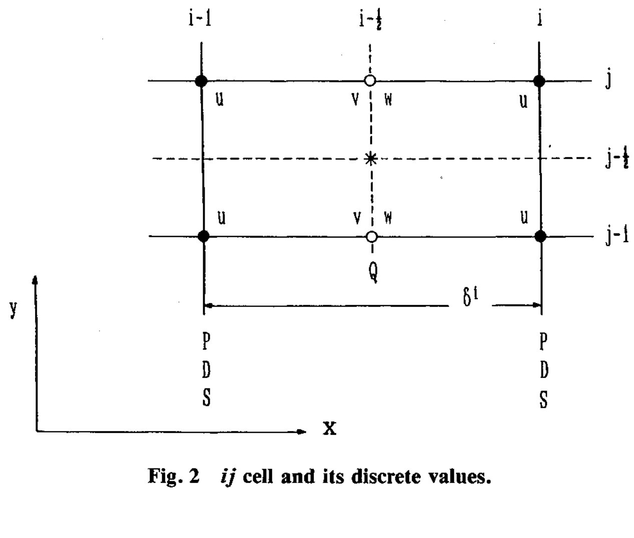


Fig. 2 ***ij* cell and its discrete values.**

and the pressure gradient function $Q(x)$ defined by

$$Q = P_x \qquad \text{(3b)}$$

is introduced for convenience.

In addition, the following upstream and downstream conditions apply to the leading order[7]:

$$D \to 0 \qquad \text{as } x \to -\infty \qquad \text{(4a)}$$

$$p \to \alpha \qquad \text{as } x \to \infty \qquad \text{(4b)}$$

When S is specified (for example, as zero, for a sharp corner), the problem described earlier is the analysis (direct) problem. Its solution gives the flow behavior that corresponds to the specified surface shape.

In the design (indirect) problem, we determine S such that the surface shear $\tau(x)$ given by

$$\tau = w(x, 0) \qquad \text{(5)}$$

is never less than a specified minimum value $\tau_{\min}$; i.e., $\tau \geq \tau_{\min}$. A barely attached flow is obtained when $\tau_{\min} = 0$.

All variables used earlier are scaled to be $\mathcal{O}(Re^o)$ in the lower deck and are normalized to be independent of the basic flow parameters. They are related to the corresponding physical (*) variables through affine transformations[5] of which the following are of significance to the present design problem.

$$x^* = Re^{-3/8} C^{3/8} \lambda^{-5/4} (M^2 - 1)^{-3/8} \theta^{3/2} L^* x \qquad \text{(6a)}$$

$$S^* = Re^{-5/8} C^{5/8} \lambda^{-3/4} (M^2 - 1)^{-1/8} \theta^{3/2} L^* S \qquad \text{(6b)}$$

$$\alpha^* = Re^{-1/4} C^{1/4} \lambda^{1/2} (M^2 - 1)^{1/4} \alpha \qquad \text{(6c)}$$

where a linear viscosity law is assumed and $\lambda \cong 0.332$ is the Blasius constant. The flow parameters are the Reynolds number $Re = \rho_\infty^* u_\infty^* L^* / \mu_\infty^*$, the Mach number $M = u_\infty^* / a_\infty^*$, the temperature ratio $\theta = T_B^* / T_\infty^*$, and the Chapman-Rubesin constant $C = \rho_B^* \mu_B^* / \rho_\infty^* \mu_\infty^*$, where ρ_∞^*, μ_∞^*, a_∞^*, u_∞^*, and T_∞^* are the freestream density, viscosity, sonic speed, velocity, and temperature, respectively, the subscript B denotes the corresponding surface values, and L^* is the characteristic length of the problem.

Numerical Treatment

The finite difference scheme applied to the design problem for the simultaneous solution of the lower-deck equations and determination of the augmentation function S is based on a two-sweep iterative scheme used by Napolitano et al.[6] for the solution of the lower-deck equations in the corresponding analysis problem when S is given as zero.

The computational domain in the xy plane is divided by a grid of i lines ($i = 1 \to I$) and j lines ($j = 1 \to J$) into rectangular cells. The typical ij cell is shown in Fig. 2 with the $i - ½$ and $j - ½$ lines identifying the midcell lines. Its x step size is δ^i.

The discrete values of the flow variables are denoted by $u^{i,j}$, $v^{i-½,j}$, $w^{i-½,j}$, D^i, P^i, $Q^{i-½}$, and S^i, indicating the locations to which they belong. For clarity, the locations of the discrete values of the ij cell are shown in Fig. 2.

The scheme implements an iteration cycle that involves two sweeps. In the first sweep, we solve, for u, v, w, Q, and S, Eqs. (1) with prescribed displacement function $\underline{D}$ where, as a convention, the underscores mark values obtained from previous calculations. Eqs. (1a–1c) are centered at the centroid of the ij cell. Central difference approximations in terms of the discrete values of the cell are applied to all terms except for u and u_y appearing in Eqs. (1b) and (1c), which are expressed by central differences in terms of $u(i - ½, j)$ and $u(i - ½, j - 1)$. Then, $u(i - ½, j)$ for $j = 1 \to J$ is expressed in Eq. (1b) as

$$u^{i,j} - ½\delta^i u_x(i - ½, j) \qquad \text{(7a)}$$

and in Eq. (1c) as

$$u^{i-1,j} + \tfrac{1}{2}\delta^i u_x(i - \tfrac{1}{2}, j) \tag{7b}$$

To evaluate $u_x(i - \frac{1}{2}, j)$, we use the first-order approximation $(u^{i-1,j} - u^{i-2,j})/\delta^{i-1}$ maintaining, at the same time, the second-order accuracy of the procedure. This can be done as long as u_x is continuous all through to the $(i - 2)$ line. If u_x has a discontinuity at the $(i - 1)$ line due to a discontinuous behavior of B_x, then $u_x(i - \frac{1}{2}, j)$ is evaluated by its central difference approximation $(\underline{u}^{i,j} - \underline{u}^{i-1,j})/\delta^i$. When $i = 2$, we set $u_x(i - \frac{1}{2}, j)$ equal to zero as implied by condition (1h).

This finite difference representation avoids averaging in the x direction and, thus, produces nonwiggling augmentation functions. It can be looked at as a noniterative version of the three-point-backward representation that has proven successful in other similar applications.[8,9] A version of the scheme that is first-order accurate in the x direction can be obtained by setting $u_x(i - \frac{1}{2}, j) = 0$ in (7a) and (7b).

At each i line ($i = 2 \rightarrow I$), we arrive at $3J - 3$ linear difference equations of the form ($j = 2 \rightarrow J$)

$$(v^{i-\frac{1}{2},j} - v^{i-\frac{1}{2},j-1}) + k^{i,j}(u^{i,j} + u^{i,j-1}) = K^{i,j} \tag{8a}$$

$$(u^{i,j} - u^{i,j-1}) + l^{i,j}(w^{i-\frac{1}{2},j} + w^{i-\frac{1}{2},j-1}) = L^{i,j} \tag{8b}$$

$$(w^{i-\frac{1}{2},j} - w^{i-\frac{1}{2},j-1}) + n_1^{i,j}v^{i-\frac{1}{2},j} + n_2^{i,j}v^{i-\frac{1}{2},j-1} + N_1^{i,j}Q^{i-\frac{1}{2}} = N_2^{i,j} \tag{8c}$$

where k, K, l, L, n_1, n_2, N_1, and N_2 are known coefficients.

Conditions (1d–1g) give four more equations

$$u^{i,1} = 0, \qquad v^{i-\frac{1}{2},1} = 0 \tag{8d, 8e}$$

$$w^{i-\frac{1}{2},J} = 1, \qquad u^{i,J} = y^J - \underline{D}^i + \underline{B}^i + \sigma^i \tag{8f, 8g}$$

where

$$\sigma = B - \underline{B} = S - \underline{S} \tag{9}$$

is a correction to $\underline{B}$ or $\underline{S}$ to be determined in the present cycle.

Equations (8) are used to march from the $(i - 1)$-line to the i-line starting with $i = 2$ and using condition (1h) to provide the necessary initial values at the first line. They comprise a set of $3J + 1$ algebraic equations involving the $3J + 2$ unknowns $u^{i,j}$, $v^{i-\frac{1}{2},j}$, $w^{i-\frac{1}{2},j}$ (for $j = 1 \rightarrow J$), $Q^{i-\frac{1}{2}}$, and σ^i.

The requirement of specified minimum surface shear provides the remaining equation. To achieve this, we exploit the linearity of Eqs. (8). Each unknown u, v, w, or Q is written as a linear combination according to

$$(\ ) = (\tilde{\ }) + \sigma^i(\hat{\ }) \tag{10}$$

This makes it possible to decompose the set of Eqs. (8) for the ( ) into two sets that are free from σ^i. The set for the $(\tilde{\ })$ would be identical to Eqs. (8) but with condition (8g) replaced by

$$\tilde{u}^{i,J} = y^J - \underline{D}^i + \underline{B}^i \tag{8$\tilde{g}$}$$

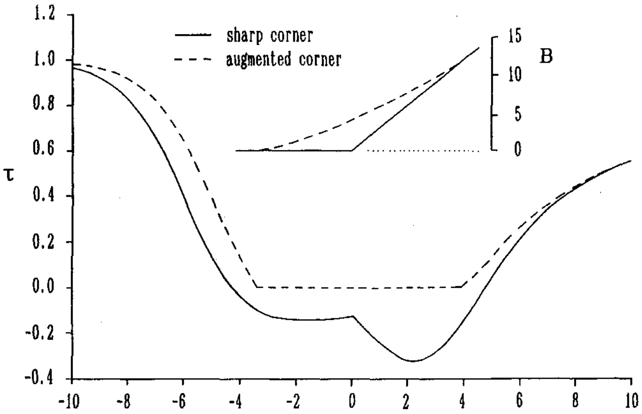
whereas the set for the $(\hat{\ })$ would have the right-hand sides of Eqs. (8) replaced by zeros except for condition (8g), which would be replaced by

$$\hat{u}^{i,J} = 1 \tag{8$\hat{g}$}$$

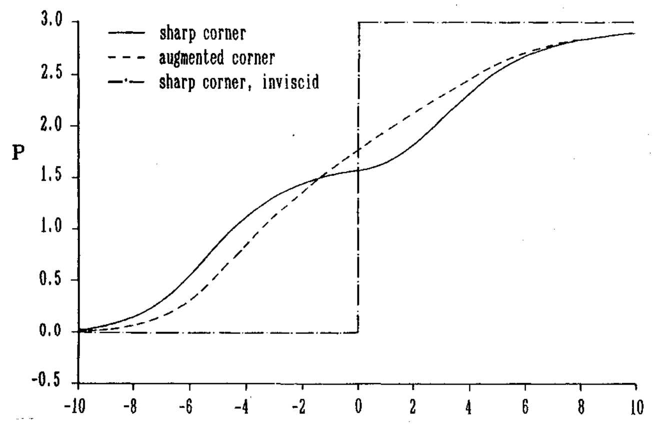
The procedure for determining σ^i is based on the observation that the ( ) problem with $\sigma^i = 0$ is identical to the $(\tilde{\ })$ problem. It goes as follows: We solve the $(\tilde{\ })$ problem. If $\tilde{w}^{i-\frac{1}{2},1} \geq \tau_{\min}$, we set $\sigma^i = 0$, take the ( ) equal to the $(\tilde{\ })$, and proceed to the $(i + 1)$ line. If, on the other hand, $\tilde{w}^{i-\frac{1}{2},1} < \tau_{\min}$, we solve the $(\hat{\ })$ problem. Enforcement of the requirement $w^{i-\frac{1}{2},1} = \tau_{\min}$ and use of relation (10), then give

$$\sigma^i = (\tau_{\min} - \tilde{w}^{i-\frac{1}{2},1})/\hat{w}^{i-\frac{1}{2},1} \tag{11}$$

With σ^i known, the $(\tilde{\ })$ and $(\hat{\ })$ can be superimposed according to relation (10) to give the ( ) needed to proceed to the $(i + 1)$ line. Furthermore, S and B can be obtained using Eqs. (9). The first sweep ends when $i = I$.



a) Surface shape B and surface shear τ



b) Pressure P

Fig. 3 Typical case, $\alpha = 3$.

In the second sweep, we solve, for D and P, the interaction law (3a) simultaneously with the relaxation relation

$$D - \underline{D} = r(P_x - \underline{Q}) \tag{12}$$

and conditions (4a) and (4b), where r is a relaxation parameter. Equations (3a) and (12) are centered at the $(i - \frac{1}{2})$ lines, and central difference representations are used. This leads to $2I - 2$ algebraic equations that are supplemented with the conditions $D^1 = 0$ and $P^I = \alpha$ as approximations to the asymptotic conditions (4a) and (4b), and are solved for D^i and P^i ($i = 1 \rightarrow I$).

The two-sweep iteration cycle is repeated until convergence is reached with relation (12) approaching the trivial identity ($0 = 0$).

It is noted that, by surpassing the special treatment used in the first sweep when $\tilde{w}^{i-\frac{1}{2},1}\tau_{\min}$, we end up with the procedure for solving the analysis problem. The determination of the augmentation function is, thus, integrated in this procedure in such a way that it does not require a new iteration loop. It turns out that, because separation is prevented, convergence of the design problem is faster than that of the analysis problem. However, it is found that the resulting augmentation has a bumpy front. This is attributed to the leading

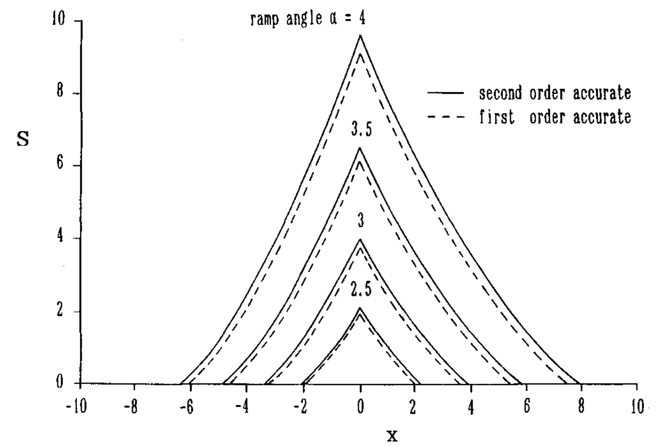
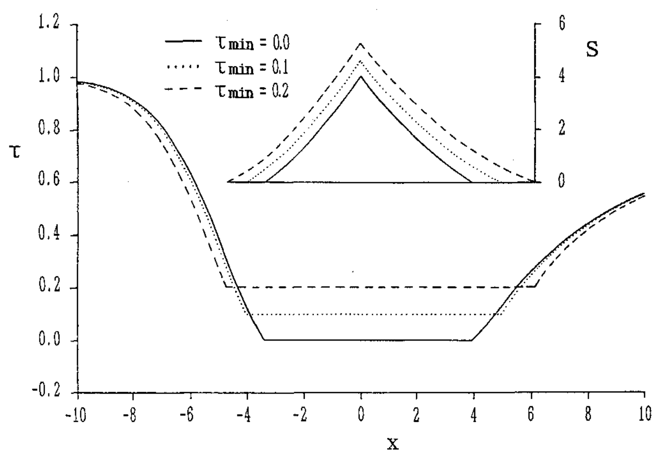
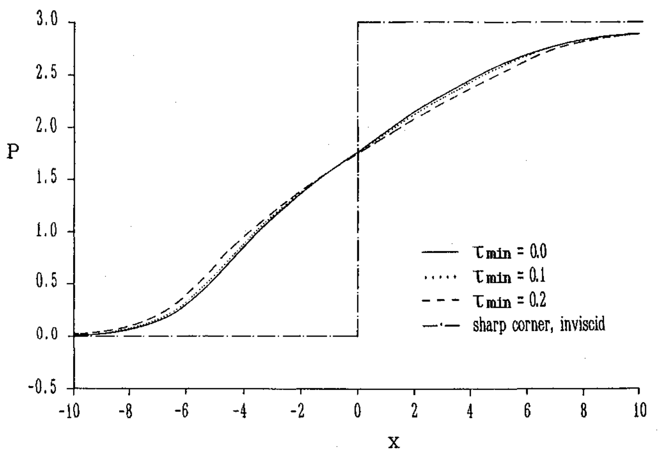


Fig. 4 Augmentation function S for barely attached flow.



a) Augmentation function S and surface shear τ



b) Pressure P

Fig. 5 Effect of the margin of safety $\tau_{\min}$.

edge of the augmentation being noncoincident with an i line. Thus, u_x has a discontinuity that is not handled properly. To avoid this irregularity, three new i lines are implanted and moved around iteratively until one of them coincides with the leading edge of the augmentation. The same adjustment is applied to the trailing edge of the augmentation, to unify the treatment. The first-order-accurate version of the scheme mentioned earlier is expected—and has proved—to be free from this difficulty, needing, therefore, no adjustments.

Results

All results given in this section are obtained using the same computational domain ($-14 \le x \le 18$, $0 \le y \le 14$). In the x direction, a step size of 0.4 is used for the intervals $-14 \le x \le -12$ and $12 \le x \le 18$, and a step size of 0.3 is used in the interval $-12 \le x \le 12$, so that $I = 101$. (The implanted i lines are not counted.) In the y direction, starting from $y = 0$, step sizes of 0.1, 0.2, 0.3, 0.4, and 0.5 are used for 4, 6, 8, 10, and 12 intervals, respectively, so that $J = 41$. This use of variable step size is easy to handle with the current box-like representation and is intended to give good resolution. The initial guess for D is taken to be the surface for the sharp corner given by Eq. (2a). The suitable value for the relaxation parameter r is found to be 6, and convergence is considered reached when the average error in D becomes less than 10^{-4}.

As a typical case, we choose the corner of ramp angle $\alpha = 3$. Two sets of solutions are obtained; one for the separated flow of the analysis problem when the corner is sharp, and the other for the barely attached flow ($\tau_{\min} = 0$) of the design problem with augmented corner. Figure 3a shows the surface shape B and the surface shear τ in both problems, and Fig. 3b shows the corresponding pressure distributions as well as the inviscid pressure distribution $P = \alpha H(x)$ for a sharp corner. It is noted that the sharp corner produces a sizable separation bubble extending over the distance $-4.33 \le x \le 4.71$. The augmentation that prevents this separation extends over the shorter distance $-3.38 \le x \le 3.94$. This result is believed to be a manifestation of the upstream influence detected by the lower-deck equations and captured, in a proper manner, by the iterative scheme. For, an important feature of the scheme is that the separation in the analysis problem and the augmentation in the design problem both start at the corner point and spread upstream and downstream to their limits with the progress of the iteration cycles. Away from the corner, the surface shear τ and the pressure P agree in both separated and attached flows. Around the corner, the obvious qualitative difference in τ is accompanied by another qualitative difference in P. The attached flow does not develop a pressure plateau as does the separated flow. Quantitatively, preventing separation by augmentation brings the pressure closer to the lossless inviscid distribution, signifying a savings of energy.

The augmentation functions that barely prevent separation for ramp angles of 2.5, 3, 3.5, and 4 are shown in Fig. 4. As the designer would expect, the larger the angle, the larger the size of the augmentation in both the x and y directions. Included, also, are the corresponding results obtained by the first-order-accurate version of the scheme, which represents a fast and practical alternative that produces results in fair agreement with the adjusted results of the second-order-accurate version.

As it is dangerous to carry out a design on the basis of the borderline condition $\tau_{\min} = 0$, a margin of safety is introduced by taking $\tau_{\min} > 0$. The results of applying values of $\tau_{\min} = 0$, 0.1, and 0.2 to the typical case $\alpha = 3$ are presented in Figs. 5a and 5b. It is noted that, as $\tau_{\min}$ increases, a larger augmentation is needed. A greater loss of energy is also expected because the minimum value of the surface shear $\tau_{\min}$ is raised and the region it acts on extends further. This energy loss is translated to a tendency of the pressure distribution to move away from the inviscid distribution. A small value of the margin of safety $\tau_{\min}$ would, therefore, be recommended.

Conclusion

Since its formulation in 1969, the triple-deck theory has been put to extensive and exhaustive application to the analysis problem of finding the flow description that corresponds to a given configuration.

The present paper is believed to present the first application of the triple-deck theory to a design problem. An augmented compression corner is designed, with different values of margin of safety, so that supersonic flow separation is not encountered. The trend of the results agrees, in all details, with the designer's intuition.

The numerical scheme in use deserves a special notice. In particular, the first-sweep marching procedure from an i line to the next is carried out in a way that makes it very efficient, computationally. Although it is second-order accurate in the x direction, it does not use the Crank-Nicholson representation and thus avoids averaging and its consequence of numerical oscillations in regions of fast changes. It, rather, relies on a three-point-backward representation not only of the convection terms, as usually done,[8,9] but also of the diffusion term, leading, contrary to the usual, to a march that is neither iterative nor requiring storage of any of the (x, y)-dependent variables u, v, and w. All we need to store are the x-dependent variables D, P, Q, and S. This is possible because of the discretization layout which assigns the discrete values of u, v, and w to the points (i, j), $(i - \frac{1}{2}, j)$, and $(i - \frac{1}{2}, j)$, respectively.

The use of a two-point representation in the y direction is a mere convenience, making the handling of the surface shear much easier. For those who would rather use a three-point representation, the momentum equation in the form [Eqs. (1a-1c) combined]

$$u_{yy} - uu_x - u_y v = Q \tag{13}$$

is centered at the point $(i - \frac{1}{2}, j)$, and u_x, v, and Q are allowed the obvious representations $(u^{i,j} - u^{i-1,j})/\delta^i$, $v^{i-\frac{1}{2},j}$, and $Q^{i-\frac{1}{2}}$. For u_{yy}, u, and u_y, central difference representations in terms of $u(i - \frac{1}{2}, j + 1)$, $u(i - \frac{1}{2}, j)$, and $u(i - \frac{1}{2}, j - 1)$ are used, then $u(i - \frac{1}{2}, j)$, for $j = 1 \rightarrow J$ is expressed as in (7a) when u_{yy} is concerned, and as in (7b) when u and u_y are concerned.

Moreover, for those[8,9] who would rather dispense with the normal velocity v in favor of a stream function ψ that is defined by

$$v = -\psi_x \tag{14a}$$

with Eq. (1a) replaced by

$$u = \psi_y \tag{14b}$$

an equivalent formulation would be obtained if ψ were assigned discrete values at the points (i, j), Eqs. (14a) and (14b) were centered at the points $(i - \frac{1}{2}, j)$ and $(i, j - \frac{1}{2})$, respectively, and central differences were used.

Acknowledgment

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